mination of the surfacing velocity for X > 0.8. This shortcoming is eliminated with the introduction of the new characteristic dimension, and system (8)-(11) is changed to the following form:

$$\mathrm{Fr}(\Pi/2)=\begin{cases} Z\,\mathrm{Bo}(\delta) \ \text{at} \ Z\leqslant 1;\ \mathrm{Bo}(\delta)<\mathrm{Bo}_2, & (17)\\ Z\,\mathrm{Bo}_2 \ \text{at} \ Z\leqslant 1;\ \mathrm{Bo}(\delta)>\mathrm{Bo}_2, & (18)\\ \mathrm{Bo}(\delta) \ \text{at} \ Z>1;\ \mathrm{Bo}(\delta)<\mathrm{Bo}_2, & (19)\\ \mathrm{Bo}_2 \ \text{at} \ Z>1;\ \mathrm{Bo}(\delta)>\mathrm{Bo}_2, & (20)\end{cases}$$

where $\mathrm{Bo}_2 = A_2$, $Z = 6.2\sqrt{2\delta/\Pi}$.

Here, A_2 becomes the boundary value of the number $\mathrm{Bo}(\delta)$, with the effect of surface forces diminishing above this value (if condition (5) is satisfied, of course).

NOTATION

ρ', ρ'', density of liquid and gas; σ, surface tension; g, acceleration due to gravity; D_0, diameter of pipe, D_1 and D_2, external and internal diameters of annular channel; b, δ, width and clearance of rectangular channel; D_{sh}, diameter of shell of rod bundle; U_∞, limiting plug surfacing velocity; l, linear dimension; $\mathrm{Bo}(l) = l/\sqrt{\sigma/g(\rho' - \rho'')}$, Bond number.

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THEORY OF GASLIFTS

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On the basis of a simplified model, a new method is proposed for the analysis of unsteady-state gaslifts, and instabilities of steady-state gaslift processes are demonstrated.

A considerable part of the petroleum which is produced is recovered by the use of various types of gaslifts for exploiting wells, and the importance of this method in the total volume of petroleum produced shows a clear tendency to increase. In this connection, the problem of optimizing gaslifts is becoming particularly important, since this is related to increasing the production of wells and decreasing their capital and operating costs; the solution of this problem is not possible without the effective modeling of the processes occurring in gaslifted wells. However, the existing methods of modeling and calculation of these processes are unsuitable for analyzing the significantly unsteady-state phenomena which occur in gaslifts. In addition, even under steady-state conditions they are not well adapted to explaining the distributions of the gas–liquid mixture in the ascending column of the well. In fact, these methods are based on semiempirical considerations of the steady-state regime only [1], and in principle they do not extend beyond the models proposed as much as a generation ago [2].

Under the conditions occurring in practice the gas lift process often appears to be unsteady-state in nature. Unsteadiness occurs in the process of starting up a well, and may also be introduced when the gaslifts are organized to operate batchwise [3]. In addition, the steady-state regime sometimes proves to be unstable, which leads ultimately to the generation of self-excited oscillations [4]. The author knows of only a single formalized approach

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towards describing the conditions under which self-excited oscillations arise (see [5]), but unfortunately this is completely unable to deal with the physical essentials of the problem. The first step in the modeling of a gaslift must therefore definitely consist of generalizing the known models and developing the necessary methods to make it possible to consider both the stability of steady-state gaslift processes and the properties of unsteady-state gaslifts. This is dealt with in the present paper.

BASIS OF THE MODEL

Let us consider a vertical tube which is hydraulically connected at the bottom of the well and with a gas-filled space around the tube. Gas is fed into this space from a compressor at a mass flow rate G, and passes into the tube through a valve placed at a height h_c above the bottom with a mass flow rate G_g. For simplicity a hydraulic connection between the outside space around the tube and the bottom zone is assumed not to exist. Liquid passes from the formation into the well bottom and then into the ascending tube with a mass flow rate G_f. Such a system obviously consists of the following interacting subsystems: the compressor, valve, well bottom, the space surround the tube which is filled with gas, the ascending tube which is filled with gas-liquid mixture, and the formation containing the liquid. The first three subsystems can be regarded usefully as systems with localized parameters. The same is also true of the fourth subsystem if the weight of the gas and the time of propagation of sonic perturbations along the space outside the tube are neglected. However, the last two of the subsystems which have been listed above represent systems with distributed parameters, which seriously complicates the analysis.

In the initial stages of the investigation it is natural to consider these distributed subsystems separately, since in order to explain the principal aspects of the problem it is desirable to simplify as much as possible the purely computational part, which implies making a series of quite restrictive assumptions. In the present paper attention is concentrated on analyzing the effects of one of the subsystems with distributed parameters (the ascending pipe with the gas-liquid mixture), while the second such subsystem (the porous formation containing the liquid) is described along with the well bottom and the part of the tube filled with liquid only as a system with localized parameters by using a relationship which follows heuristically from Darcy's theory and which is also assumed in [2]:

$$Q_{f*} = \alpha(p^* - p_*),\ p^* = p_\infty - \rho_f g h_c,\ G_f = \rho_{f*} Q_{f*}, \tag{1}$$

where the coefficient α characterizes the interaction of the well bottom with the formation.

The characteristics of the gas valve are represented in the form:

$$Q_{g*} = F(P - p_*),\ G_g = \rho_{g*} Q_{g*}. \tag{2}$$

The characteristics of the compressor determine the mass flow rate of gas which is injected into the space around the tube in the form of some function of the pressure in this space. The form of this function is not important in principle, so that here it is assumed simply that

$$G = G_0 = \text{const}. \tag{3}$$

Finally, the characteristics of the space surrounding the tube should determine the relationship between the changes in the total mass of gas and its pressure, which depends on the nature of the processes of compression and expansion. For a polytropic process,

$$P/\rho^\gamma = \text{const},\ \rho = MP/R\theta, \tag{4}$$

where γ is equal to unity or to the adiabatic exponent for the particular gas under isothermal or adiabatic conditions, respectively.

The expansion of the gas rising in the tube will also be described by means of the relationship for a polytropic process:

$$P/\rho_g^{\gamma_g} = \text{const},\ \rho_g = Mp/R\theta_g, \tag{5}$$

which is analogous to (4), but with an exponent γ_g which is different, generally speaking.

The distributed parameters characterizing the ascending tube depend primarily on the flow regime of the gas—liquid mixture in it, and in view of the specificity of such flows (see, for instance, [6]), they cannot be specified completely uniquely. In order to eliminate this difficulty and simplify the calculations, the presence of slip between the phases in the tube is ignored throughout the treatment here, as is the pressure drop in the tube caused by viscous friction and the acceleration on the mixture. The first assumption is approximately valid, strictly speaking, if the rate of rise of the gas bubbles with respect to the mean velocity of the mixture is small, so that the velocities of the two phases can be regarded as being the same. To a certain extent the second assumption is justified by the fact that in practice the main pressure drop along the well line is caused usually by the weight of the volume of mixture being driven along it. The possibilities of generalizing to escape from these simplifying assumptions will become clear from the presentation below.

To make the problem well defined, the weight of the gas in the ascending pipe will also be neglected and it will be assumed that the liquid does not contain dissolved gas. The pressure drop in the pipe is then related only to the weight of the column of liquid existing in it, where the liquid is assumed to be incompressible, and the gas content is determined from the relationship between the masses of gas and liquid being fed to the "inlet" section (located at a distance h_c above the bottom) through the valve and from the well bottom, respectively.

THE DEFINING EQUATIONS

In order to analyze the motion of portions of gas and liquid fed into the inlet zone of the ascending pipe it seems natural to use the method of Lagrange. At the moment of time t_0 assume that the gas and liquid pass into the tube with volumetric rates $Q_{g*}(t_0)$ and $Q_{f*}(t_0)$; then after a time Δt_0, volumes $Q_{g*}(t_0)\Delta t_0$ and $Q_{f*}(t_0)\Delta t_0$ of the gas and the liquid will have passed into the pipe. At some moment of time $t > t_0$ this portion of the mixture will have risen to a height $z(t_0, t)$ above the inlet cross section. Since the volume of the portion of liquid does not change, as a result of its incompressibility, while the volume of the portion of gas increases due to its expansion, the length of the section of tube occupied by these volumes at the moment t is given by:

$$\Delta l(t_0, t) = \frac{1}{S}\left[\frac{\rho_{g*}(t_0)}{\rho_g(t_0, t)} Q_{g*}(t_0) + Q_{f*}(t_0)\right]\Delta t_0. \tag{6}$$

The Lagrangian coordinate $z(t_0, t)$ can be determined by evaluating the length of the lower part of the tube filled by the gas and liquid introduced in the course of the time interval (t_0, t). By using the expression (6) and replacing t_0 by $\tau \geqslant t_0$ and Δt_0 by $\Delta\tau$, carrying out summations over various intervals $\Delta\tau$, and converting from sums to the corresponding integrals at $\Delta\tau \to 0$, it is found that

$$z(t_0, t) = \frac{1}{S}\int_{t_0}^{t}\left[\frac{\rho_{g*}(\tau)}{\rho_g(\tau, t)} Q_{g*}(\tau) + Q_{f*}(\tau)\right] d\tau. \tag{7}$$

The pressure $p(t_0, t)$ at the moment of time t and at the height $z(t_0, t)$ above the inlet cross section of the inlet tube is less than the inlet pressure $p_*(t)$ at this moment by the value of the weight of liquid in the lower part of the tube of height $z(t_0, t)$ referred to the area of its transverse cross section, i.e.,

$$p(t_0, t) = p_*(t) - \frac{\rho_f g}{S}\int_{t_0}^{t} Q_{f*}(\tau)\, d\tau. \tag{8}$$

According to (5), the ratio of densities in Eqs. (6) and (7) becomes

$$\frac{\rho_{g*}(\tau)}{\rho_g(\tau, t)} = \left[\frac{p_*(\tau)}{p(\tau, t)}\right]^{1/\gamma_g}. \tag{9}$$

Suppose that the portion of mixture fed into the tube at the moment t_0 reaches the outlet of the tube (the "daylight" surface) at the moment $t = t_0 + T(t_0)$. It is clear that $z(t_0, t) = H$, and the pressure $p(t_0, t) = p_0$, where p_0 is the specified pressure (for instance, atmospheric pressure) at the outlet. A system of integral equations with lagging arguments is therefore obtained from (7) and (8) (the symbol t is introduced instead of t_0):

$$\begin{gathered} p_*(t + T(t)) = p_0 + \frac{\rho_f g}{S}\int_{t}^{t+T(t)} Q_{f*}(\tau)\, d\tau, \\ H = \frac{1}{S}\int_{t}^{t+T(t)}\left[\frac{\rho_{g*}(\tau)}{\rho_g(\tau, t+T(t))} Q_{g*}(\tau) + Q_{f*}(\tau)\right] d\tau, \end{gathered} \tag{10}$$

and this system is closed by the following relationship which follows from (8) and (9):

$$\frac{\rho_{g*}(\tau)}{\rho_g(\tau,\ t+T(t))}=\left\{p_*(\tau)\left[p_*(t+T(t))-\frac{\rho_f g}{S}\int\limits_{\tau}^{t+T(t)}Q_{f*}(\tau')\,d\tau'\right]^{-1}\right\}^{1/\gamma_g}. \tag{11}$$

The system of equations (10) together with the closing relationship (11) serve for determining the two unknown functions, the pressure $p_*(t)$ at the inlet to the tube and the time fo ascent T(t) of the portion of mixture introduced at the moment of time t.

By standard methods it is possible to convert from the Lagrangian method of describing the motion in the tube using Eqs. (7)-(11) to the Eulerian system. With this objective in mind we first define the pressure p(t, x) as a function of time and the longitudinal coordinate, which are regarded as independent variables. In parametric form it is found that

$$p^e(t,\ x)=p(t_0,\ t),\ x=z(t_0,\ t), \tag{12}$$

where t_0 plays the part of a parameter. From Eq. (9) it is therefore easy to also determine the Eulerian density of the gas in the ascending tube, $\rho^e(t, x)$. Then by differentiating Eq. (7) with respect to time, and taking into account the formula following from (5) and (8),

$$\frac{\partial\ln\rho_g(\tau,\ t)}{\partial t}=\frac{1}{\gamma_g p(\tau,\ t)}\left[\frac{dp_*}{dt}-\frac{\rho_f g}{S}Q_{f*}(t)\right],$$

an expression is found for the velocity of the mixture in the tube:

$$u(t,\ x)=\frac{1}{S}\left[\frac{\rho_{g*}(t)}{\rho_g(t,\ t)}Q_{g*}(t)+Q_{f*}(t)\right]-\frac{1}{\gamma_g S}\left[\frac{dp_*}{dt}-\frac{\rho_f g}{S}Q_{f*}(t)\right]\int\limits_{t_0}^{t}\frac{\rho_{g*}(\tau)\,Q_{g*}(\tau)}{\rho_g(\tau,\ t)\,p(\tau,\ t)}\,d\tau. \tag{13}$$

The corresponding expression for the gas content $\varphi(t, x)$ is obvious from a consideration of the physical significance of Eq. (6):

$$\varphi(t,\ x)=1-Q_{f*}(t_0)\left[\frac{\rho_{g*}(t_0)}{\rho_g(t_0,\ t)}Q_{g*}(t_0)+Q_{f*}(t_0)\right]^{-1}. \tag{14}$$

In expressions (13) and (14) the quantity t_0 is primarily regarded as a parameter which depends on t and x in accordance with Eq. (12).

It is important that the relationship and Eqs. (6)-(14) are suitable for describing within the framework of the simplified model being considered not only steady-state but also arbitrary unsteady-state gaslift processes. However, the methods of effective solution of the system of integral equations (10) have clearly been insufficiently developed, which indicates the need to pose and investigate a series of new mathematical problems.

STEADY-STATE CONDITIONS

Under steady-state conditions the quantities $Q_{g*\varepsilon}$, Q_{f*s}, and p_{*s} are independent of time, and the integrals which appear in the formulas given above are easy to evaluate. As a result, the first equation in (10) assumes the form:

$$p_{*s}=p_0+\rho_f g\frac{Q_{f*s}}{S}T_s, \tag{15}$$

and making use of (11) and (15), the second equation in (10) becomes

$$H=\frac{p_{*s}-p_0}{\rho_f g}+\frac{\gamma_g}{\gamma_g-1}\,\frac{p_{*s}}{\rho_f g}\,\frac{Q_{g*s}}{Q_{f*s}}\left[1-\left(\frac{p_0}{p_{*s}}\right)^{\frac{\gamma_g-1}{\gamma_g}}\right]. \tag{16}$$

Under isothermal conditions (γ_g = 1), we find instead of (16):

$$H=\frac{p_{*s}-p_0}{\rho_f g}+\frac{p_{*s}}{\rho_f g}\,\frac{Q_{g*s}}{Q_{f*s}}\ln\frac{p_{*s}}{p_0}. \tag{17}$$

Under steady-state conditions $Q_{g*s} = G_0/\rho_{g*s}$, and Q_{f*s} can be expressed in terms of p_{*s} from (1). It is therefore clear that (16) or (17) makes it possible to find p_{*s} as a function of ρf, α, and the system parameters H, p^*, p_0, and G_0. From the known value of the pressure p_{*s} it is then easy to proceed to find Q_{f*s} and P_s from (1) and (2) and then the value of T_s from (15).

The equation for t_0 from (12) has the following form under steady-state conditions:

$$x = \frac{Q_{f*s}}{S}(t - t_0) + \frac{\gamma_g}{\gamma_g - 1}\frac{p_{*s}}{\rho_f g}\frac{Q_{g*s}}{Q_{f*s}}\left\{1 - \left[1 - \frac{\rho_f g Q_{f*s}}{p_{*s}S}(t - t_0)\right]^{\frac{\gamma_g - 1}{\gamma_g}}\right\}, \tag{18}$$

or, with $\gamma_g = 1$,

$$x = \frac{Q_{f*s}}{S}(t - t_0) - \frac{p_{*s}}{\rho_f g}\frac{Q_{g*s}}{Q_{f*s}}\ln\left[1 - \frac{\rho_f g Q_{f*s}}{p_{*s}S}(t - t_0)\right]. \tag{19}$$

From this it can be seen that under steady-state conditions the parameter t_0 has the form $t - f(x)$. If $t - t_0 = T_s$ $(x = H)$, then taking into account the representation for T_s which follows from (15), equations (18) and (19) reduce to equations (16) and (17), respectively.

The Eulerian representations for the pressure, velocity, and gas content under steady-state conditions are obtained directly from (12)-(14):

$$p^e(t, x) = p_{*s} - (p_{*s} - p_0) f(x)/T_s,$$

$$u(t, x) = \frac{1}{S}\left[\left(\frac{p_{*s}}{p^e(t, x)}\right)^{1/\gamma_g} Q_{g*s} + Q_{f*s}\right], \tag{20}$$

$$\varphi(t, x) = 1 - Q_{f*s}\left[\left(\frac{p_{*s}}{p^e(t, x)}\right)^{1/\gamma_g} Q_{g*s} + Q_{f*s}\right]^{-1},$$

where the function $f(x) = t - t_0$ is determined from (18) or (19).

It should be noted that the applied importance of the results which have been obtained is limited, since above the slip between the phases was ignored, as well as the consumption of energy for overcoming friction and accelerating the liquid mass in the ascending tube. Thus, for example, Eq. (16) or (17) has a single positive root satisfying the inequalities $p_{*s} < p^*$ and $p_{*s} < p_0 + \rho_f gH$, at any value of the parameter, i.e., broadly speaking, ascent of the liquid under steady-state conditions appears to be possible regardless of how small the gas flow rate is. This conclusion, of course, is not correct, since at small velocities of the mixture it is impossible in principle to neglect the effects of slip between the phases. However, these results are quite adequate for attaining the main objectives of the present work, namely, demonstrating the general procedures for obtaining and investigating the stability of the steady-state regime.

STABILITY OF THE STEADY-STATE REGIME

Suppose that as a result of random causes the steady-state value P_s of the pressure in the space around the tube at the moment of time t_0 varies by some small amount $(\delta P)_0$, i.e., that at $t \geqslant t_0$, $P(t) = P_s + \delta P(t)$, $\delta P(t_0) = (\delta P)_0$. Correspondingly, the gas density in this space is expressed in the form

$$\rho(t) = \rho_s + \delta\rho(t),\ \delta\rho = \frac{\rho_s}{\gamma}\frac{\delta P}{P_s}.$$

This perturbation in the pressure leads to a change in the relationship between the volumes of the gas and liquid being fed into the tube, and hence in the total weight of liquid in the tube and in the value of the inlet pressure. Therefore at $t \geqslant t_0$ it is found that $p_*(t) = p_{*s} + \delta p_*(t)$, from which it is obvious that $\delta p_*(t_0) = 0$.

The equation for the conservation of the mass of gas in the space around the tube has the form

$$d(\rho V)/dt = G_0 - \rho_{g*} Q_{g*}.$$

Hence, taking Eq. (3)-(5) into account, and assuming for simplicity that the gas temperature does not vary on passing through the valve (i.e., $\theta = \theta_g$ and $P_s/\rho_s = p_{*s}/\rho_{g*s}$), then to an accuracy of terms of the first order with respect to the perturbation the following equation is obtained

$$V\frac{d}{dt}\delta P = -\gamma p_{*s}F_s'\delta P + \left(\gamma p_{*s}F_s' - \frac{\gamma}{\gamma_g}F_s\right)\delta p_*, \tag{21}$$

where

$$F_s = F(y),\ F_s' = dF(y)/dy \text{ with } y = P_s - p_{*s}.$$

The second equation for $\delta P(t)$ and $\delta p_*(t)$ can be found by bearing in mind that the difference $p_*(t) - p_0$ is equal as usual to the weight of all the liquid in the ascending tube referred to the area of the tube cross section. Changes of this difference compared with its steady-state value arise as a result of two factors. In the first place, after a time $t - t_0$ from the moment at which the perturbation occurs a quantity of liquid enters the ascending tube which is different to the steady-state value. In the second case, because of the perturbation of the velocity of the mixture a quantity of liquid flows out of the upper and of the tube after this time which is also not equal to the quantity which would leave in the steady state. Thus,

$$\delta p_*(t) = \delta_1 p_*(t) + \delta_2 p_*(t), \tag{22}$$

where the right-hand side of the equation includes the components caused by these two factors. The concept of the first term in (22) is clear:

$$\delta_1 p_*(t) = \frac{\rho_f g}{S} \int_{t_0}^{t} \delta Q_{f*}(\tau)\, d\tau. \tag{23}$$

The concept of the second term is obtained by considering that the interval of time $t - t_0$ is sufficiently small that the analysis can be restricted to effects of the first order only with respect to the value of this time interval. During the time $t - t_0$ a portion of the mixture enters the tube which has a gas content differing from the steady-state value; this occupies the lower part of the tube whose height (taking into account Eq. (7) and retaining only terms of the first order with respect to the perturbation) is given by

$$z(t_0,\ t) = z_s(t_0,\ t) + \delta z(t_0,\ t),\ \ \delta z(t_0,\ t) = \frac{1}{S}\int_{t_0}^{t}\left\{\frac{\rho_{g*s}}{\rho_{gs}(\tau,\ t)}\delta Q_{g*}(\tau) + \delta Q_{f*}(\tau) + \delta\left[\frac{\rho_{g*}(\tau)}{\rho_g(\tau,\ t)}\right] Q_{g*s}\right\} d\tau.$$

From (8) and (9) it follows that

$$\rho_g(\tau,\ t) = \rho_{g*}(\tau) + O(t-\tau),\ \ \rho_{gs}(\tau,\ t) = \rho_{g*s} + O(t-\tau).$$

Thus, to a linear approximation with respect to $t - t_0$ it is found that

$$\delta z(t_0,\ t) = \frac{1}{S}\int_{t_0}^{t}[\delta Q_{g*}(\tau) + \delta Q_{f*}(\tau)]\, d\tau.$$

Taking into account that above the level $z(t_0,\ t)$ the state of the gas—liquid mixture in the tube is the same as in the steady-stage, then to a linear approximation with respect to $t - t_0$ and to an accuracy of therms of the first order with respect to the perturbation, the perturbation of the volume of liquid leaving the tube during the time being considered can be respresented as follows:

$$(1-\varphi_{*s}) S\delta z(t_0,\ t) = \frac{Q_{f*s}}{Q_{g*s} + Q_{f*s}} S\delta z(t_0,\ t).$$

Hence,

$$\delta_2 p_*(t) = -(1-\varphi_{*s})\frac{\rho_f g}{S}\int_{t_0}^{t}[\delta Q_{g*}(\tau) + \delta Q_{f*}(\tau)]\, d\tau. \tag{24}$$

Finally, from expression (22)-(24) it is found that

$$\delta p_*(t) = \frac{\rho_f g}{S}\int_{t_0}^{t}[\varphi_{*s}\delta Q_{f*}(\tau) - (1-\varphi_{*s})\delta Q_{g*}(\tau)]\, d\tau. \tag{25}$$

By substituting for the function under the integral in Eq. (25) in terms of $\delta P(\tau)$ and $\delta p_*(\tau)$ in accordance with (1) and (2) and differentiating with respect to time, a second linear differential equation is obtained

$$\frac{S}{\rho_f g}\ \frac{d}{dt}\delta p_* = -(1-\varphi_{*s}) F_s' \delta P + [(1-\varphi_{*s}) F_s' - \varphi_{*s}\alpha]\, \delta p_*. \tag{26}$$

The characteristic equation of the system (21), (26) has the following form (a solution of this system proportional to $\exp(\lambda t)$ is sought):

$$\lambda^2 + A\lambda + B = 0,$$
$$A = \frac{\rho_f g}{SV}\left\{\left[\gamma\frac{p_{*s}}{\rho_f g}S - (1-\varphi_{*s})V\right]F_s' + \varphi_{*s}V\alpha\right\},$$
$$B = \frac{\rho_f g}{SV}\left[\gamma\varphi_{*s}\alpha p_{*s} - \frac{\gamma}{\gamma_g}(1-\varphi_{*s})F_s\right]F_s', \tag{27}$$
$$\varphi_{*s} = Q_{g*s}(Q_{g*s}+Q_{f*s})^{-1}.$$

Let us now consider the zone of instability. Suppose that $A < 0$, i.e., that

$$\left[1-\gamma\frac{\alpha(p^*-p_{*s})+F_s}{\alpha(p^*-p_{*s})}\frac{p_{*s}S}{\rho_f gV}\right]F_s' > \frac{F_s}{p^*-p_{*s}} \tag{28}$$

(here and below φ_{*s} is expressed in terms of F_s and $\alpha(p^* - p_{*s})$ using (1), (2), and the last equation in (27)). In this case, instability sets in at any value of B. Transformation of the inequality (28) shows that instability occurs when the following inequalities are satisfied simultaneously:

$$V > \gamma\frac{\alpha(p^*-p_{*s})+F_s}{\alpha(p^*-p_{*s})}\frac{p_{*s}S}{\rho_f g},$$
$$\frac{F_s'}{F_s} < \frac{1}{p^*-p_{*s}}\left[1-\gamma\frac{\alpha(p^*-p_{*s})+F_s}{\alpha(p^*-p_{*s})}\frac{p_{*s}S}{\rho_f gV}\right]^{-1} \tag{29}$$

or the inequalities

$$V > \gamma\frac{\alpha(p^*-p_{*s})+F_s}{\alpha(p^*-p_{*s})}\frac{p_{*s}S}{\rho_f g},$$
$$\frac{F_s'}{F_s} > -\frac{1}{p^*-p_{*s}}\left[\gamma\frac{\alpha(p^*-p_{*s})+F_s}{\alpha(p^*-p_{*s})}\frac{p_{*s}S}{\rho_f gV}-1\right]^{-1}, \tag{30}$$

which are imposed on the volume of the space around the tube and on the steepness of the nonlinear characteristics of the valve, respectively. Oscillations are observed when $B > A^2/4$.

Now suppose that $A > 0$, which corresponds to the reverse inequalities (28)-(30). In this case for the onset of instability it is necessary that $B < 0$, which after some simple rearrangements leads to the inequality

$$p^* > (1+\gamma_g)p_{*s}. \tag{31}$$

When $B > 0$ the perturbations are damped, aperiodically when $B < A^2/4$, and with oscillations in the opposite case.

The inequalities (29)-(31) make it possible, in the first place, to investigate the effects of various parameters on the breakdown of the stability of the steady-state regime, and in the second place, to indicate means for preventing or in the opposite case, for exciting instabilities. For example, in the case of (31) instabilities can be avoided by increasing the height h_c at which the valve is positioned at a fixed value of p_∞.

The physical nature of the breakdown of the stability of the steady-state regime of a gaslift is analogous to that occurring in the fluidization of a disperse layer by a gas in the presence of free cavities filled with this gas [7, 8]. If the pressure drop in the stationary disperse layer is large, the escape of gas through it does not compensate for the entry of gas into the cavities from the outside, and energy accumulates in these cavities which is stored in the form of the energy of the compressed gas. Upon reaching some critical value of the pressure in the free cavities, the bed becomes fluidized, and its pressure drop falls sharply. As a result, the excess volumes of gas from the cavities escape rapidly through the layer, the bed subsides again, and then the process repeats itself again. Processes of the same type occur in gaslifts, although here the self-oscillations which occur are not at all necessarily of a relaxational nature. The analog of the pressure drop of the layer with nonlinear characteristics is in the present case the overall resistance of the valve and ascending tube. At the same time, there is an important difference: the liquid also enters the tube at the same time as the gas, and this influences the inlet pressure.

As P increases the passage of the gas into the tube from the space surrounding the tube increases, which must lead to a decrease of p_* in view of the increase in the gas content in the tube. However, simultaneously the passage of the liquid from the well bottom zone into the tube also increases, decreasing the gas content in it and preventing the increase of p_*. Thus, the interpretation of the conditions (29)-(31) for the onset of instability is not as clear in the present case as in [7, 8].

An interesting difference in the evolution of the linear perturbations of steady-state gaslift processes in the presence of instabilities from the usual picture of the development of hydrodynamic instabilities should be noted. A small perturbation usually increases until the inherent system of nonlinearitites stabilizes them at some finite level with the establishment of an ordered or chaotic secondary flow. In the present case such stabilization can occur in the linear zone as a result of its completely different cause, which is the nonlocal nature of the defining equations. In fact, the perturbation of the inlet pressure p_* is determined not so much by the perturbation of the pressure in the space around the tube at the same moment as by the value of the perturbation at an interval of time which is relatively large, i.e., there is an unusual cumulative effect. The latter can be subjected to analysis if the assumption of the smallness of $t - t_0$ is not made in the derivative of Eq. (26); in this case a considerably more complex equation is obtained which contains the time in explicit form.

NOTATION

F, nonlinear characteristic of the gas valve; f, value of $t - t_0$ in the steady-state regime; G, mass flow rate; g, accelerating force of gravity; H, length of active zone of ascending tube; h_c, height of the valve position; M, molecular weight of gas; P, p, pressures in space surrounding tube and in tube, respectively; p_∞, pressure in formation; Q, volumetric flow rate; R, gas constant; S, cross-sectional area of tube; T, time of passage of portion of mixture in tube; t, t_0, current and initial moments of time; u, velocity of mixture V, volume of space surrounding tube; x, longitudinal coordinate; z, Lagrangian coordinate of portion of mixture; α, coefficient of hydraulic resistance of the system formation — well bottom; γ, polytropic exponent; θ, absolute temperature; λ, increment of growth of perturbation; ρ, density; φ, gas content.

SUBSCRIPTS

f, g, liquid and gas in tube; s, steady-state parameters; *, inlet parameters.

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